

# Spike synchronization of a chaotic array as a phase transition

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We study how a coupled array of spiking chaotic systems synchronizes to an external driving in a short time. Synchronization means spike separation at adjacent sites much shorter than the average inter-spike interval; a local lack of synchronization is called a defect. The system displays sudden spontaneous defect disappearance at a critical coupling strength. Below critical coupling, the system reaches order at a definite amplitude of an external input; this order persists for a fixed time slot. Thus, the array behaves as an excitable system, even though the single element lacks such a property. The above features provide a dynamical explanation of feature binding in perceptual tasks.

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Temporal versus rate coding for the neural-based information has been open to debate in the neuroscience literature [1]. The electrical activity of a single neuron is measured by micro-electrodes inserted in the cortical tissue of animals [2, 3]. This activity consists of trains of action potentials or "spikes". In rate coding only the mean frequency of spikes over a time interval matters, thus requiring a suitable counting interval, which seems unfit for fast decision tasks. Temporal coding assigns importance to the precise timing and coordination of spikes. A special type of temporal coding is synchrony, whereby information is encoded by the synchronous firing of spikes of selected neurons in a cortical module.

Let an animal be exposed to a visual field containing two separate objects. Since each receptive field isolates a specific detail, one should expect a corresponding large set of different responses. On the contrary, all the cortical neurons whose receptive fields are pointing to the same object synchronize their spikes, and as a consequence the visual cortex organizes into separate neuron groups oscillating on distinct spike trains for different objects (*feature binding*) [2]. Indirect evidence of synchronization has been reached for human subjects as well, by processing the EEG (electro-encephalo-gram) data [4].

Since feature binding results from the readjustment of the temporal positions of the spikes, a plausible explanation is based on the mutual synchronization properties of chaotic oscillators; in fact, an erratic distribution of the spike occurrence prior to an applied stimulus seems mandatory for adapting to different rhythms ([5], and references therein).

In this Letter we show that spiking chaotic dynamics provides a suitable model of the feature binding process, where fast synchronization is necessary for completion of a perceptual task [2, 4]. Indeed, the conscious perception of a feature (e.g. form or motion) is believed to be due to the synchronization of the neurons of a cortical module within a limited time slot [6]; the time limitation is necessary to leave room to successive perceptions. Feature binding implies two temporal aspects [2, 4, 5], namely, i) it is associated with gamma band EEG oscillations (25 msec mean interspike separation), and ii) it lasts a few hundred milliseconds. Since we intend to model the neurons of a cortical module during a perceptual task, we

study the transient dynamics of defects in the coupled system. A defect is defined as the lack of synchronization of two adjacent sites. Defect disappearance marks the global synchronization of a modulus. The two main results of this paper are: i) above a critical coupling a chaotic array fully synchronizes within tens of interspike intervals; ii) in presence of an input, the array displays a collective recognition which lasts for a fixed time.

We focus on a model exhibiting homoclinic chaos (HC) [7] and compare it with a generic chaotic system as e.g. Rössler [8]. Let us consider an array of HC systems with nearest neighbor bidirectional coupling ruled by

$$\begin{aligned}\dot{x}_1^i &= k_0 x_1^i (x_2^i - 1 - k_1 \sin^2 x_6^i) \\ \dot{x}_2^i &= -\gamma_1 x_2^i - 2k_0 x_1^i x_2^i + g x_3^i + x_4^i + p \\ \dot{x}_3^i &= -\gamma_1 x_3^i + g x_2^i + x_5^i + p \\ \dot{x}_4^i &= -\gamma_2 x_4^i + z x_2^i + g x_5^i + z p \\ \dot{x}_5^i &= -\gamma_2 x_5^i + z x_3^i + g x_4^i + z p \\ \dot{x}_6^i &= -\beta [x_6^i - b_0 + r (f(x_1^i) + \epsilon(x_1^{i-1} + x_1^{i+1} - 2\eta^i(t)))]\end{aligned}\quad (1)$$

where  $f(x_1^i) = \frac{x_1^i}{1+\alpha x_1^i}$  and  $\eta^i(t)$  is a variable obeying the filter equation  $\dot{\eta}^i = -d(\eta^i - x_1^i)$  with  $d = 10^{-3}$ . The index  $i$  denotes the  $i$ th site position for  $i = 1, \dots, M$ . The values of parameters are:  $k_0 = 28.5714$ ,  $k_1 = 4.5556$ ,  $\gamma_1 = 10.0643$ ,  $\gamma_2 = 1.0643$ ,  $g = 0.05$ ,  $p = 0.016$ ,  $z = 10$ ,  $\beta = 0.4286$ ,  $\alpha = 32.8767$ ,  $r = 160$  and  $b_0 = 0.1032$ . Their physical meaning has been discussed in Ref. [7]. This parameter choice is by no means critical, since several successive finite chaotic windows are available. The mutual coupling consists of adding to the  $x_6$  equation on each site a function of the intensity  $x_1$  (action potential) of the neighboring oscillators. Chaos due to the homoclinic return to a saddle focus implies a high sensitivity to an external perturbation in the neighborhood of the saddle [5].

At each pseudo-period, or inter spike interval =  $ISI$ , HC yields the alternation of a regular large spike and a small chaotic background. The chaotic background is the sensitive region where the activation from the neighbors occurs, while the spike provides a suitable signal to activate the cou-

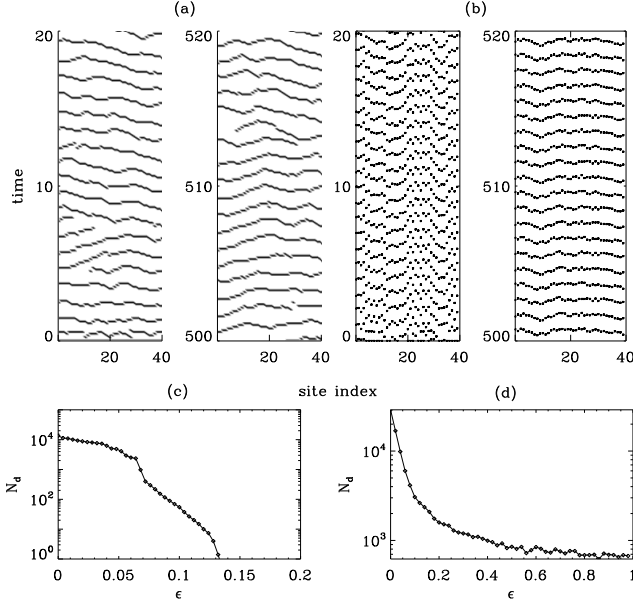


FIG. 1: Space-time plots of (a) HC systems for  $\epsilon = 0.1$  where time is in  $\langle IST \rangle$  units and (b) Rössler systems for  $\epsilon = 0.01$  where time is in  $T$  units. The degree of synchronization is characterized by the number of defects  $N_d$  estimated far beyond the initial transient: (c) 100 coupled HC; (d) 40 coupled Rössler.

pling. We use the generic attribution of HC for a large class of systems, with a saddle focus instability [9]. This includes a class B laser with feedback [7], a modified Hodgkin-Huxley model for action potentials in a neuron membrane [10] and the Hindmarsh-Rose model of generation of spike bursts [11]. In view of this high sensitivity, we expect that they synchronize not only under an external driving [12], but also for a convenient mutual coupling strength. A quantitative indicator of this sensitivity is represented by the so-called propensity to synchronization [13]. Furthermore, for an array of coupled systems, the onset of a collective synchronization was explored numerically for different values of the coupling strength  $\epsilon$ , showing that the lack of synchronization manifests itself as phase slips, that is one spike less or more compared to the adjacent site over the same time interval. In a space-time plot, space denoting the site position and time the point-like occurrence of spikes, phase slips appear as dislocations [14]. Space-wise, a tiny variation of  $\epsilon$  changes dramatically the size of the synchronized domain [14]. Here we explore the dynamical mechanisms underlying the onset of synchronization, both the spontaneous one, occurring in a coupled array above a critical  $\epsilon$  in the absence of an external input, as well as the stimulated one (semantic response) induced by an external stimulus localized at one site. Stimulated array synchronization occurs for couplings below  $\epsilon_c$ .

We analyze the interaction of neurons by taking the large amplitude spikes as 1 and the remaining chaotic and refractory background as 0. Then we define the response time  $t_r$  as the time difference between spike occurrence at neighboring

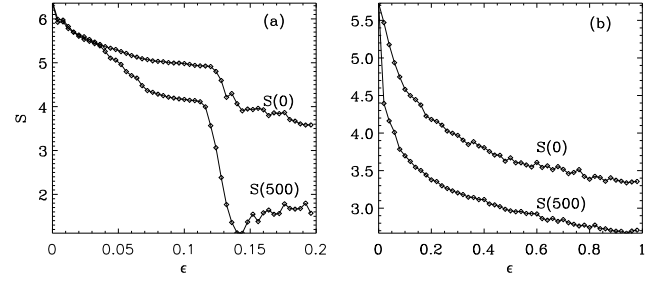


FIG. 2: The degree of synchronization of a coupled array estimated during the initial transient characterized by entropies  $S(0)$  and  $S(500)$  versus  $\epsilon$  for (a) 100 coupled HC and (b) 40 coupled Rössler.

sites starting from the first. When neurons are uncoupled, their spikes are uncorrelated and the distribution of the response times (including defects) is spread over a broad range. Increasing the coupling strength correlated clusters appear, being however characterized by a large variance in the response times and boundary appearance of defects. Indeed, a large fluctuation in the response time corresponds to a defect which interrupts a sequence of synchronized sites, representing the boundary with between two clusters. Increasing further the coupling strength we observe the transition to overall synchronization, accompanied by equal response times for all sites. The synchronization is not isochronous, as the response time distribution consists of two symmetric non zero peaks. This is due to a time lag starting from the two end sites, which obey open boundary conditions. In the case of unidirectional coupling we still have the synchronization without isochronism [15], but this time with a single  $t_r$  peak.

Spiking systems appear to be the operating units of the brain cortex [1]. In order to find out what is the peculiarity of spiking systems in the synchronization processes, we compare the spiking dynamics of HC with another chaotic system, namely Rössler. The oscillations in Rössler are characterized by a leading single frequency  $1/T$  and chaos appears in the amplitude of oscillations. Here, no spike but rather phase synchronization occurs. This implies a time code with a poorer resolution, as a spike duration in HC is less than one tenth the  $\langle IST \rangle$ , whereas phase is resolved over a sizable fraction of the period  $T$ . The dynamics is ruled by

$$\begin{aligned}\dot{x}_1^i &= -x_2^i - x_3^i + \epsilon(x_1^{i-1} + x_1^{i+1} - 2x_1^i) \\ \dot{x}_2^i &= x_1^i + ax_2^i \\ \dot{x}_3^i &= b + x_3^i(x_1^i - c)\end{aligned}\quad (2)$$

where  $a = 0.15$ ,  $b = 0.2$  and  $c = 10$ . We analyze the Rössler systems in a similar manner: when the oscillation crosses the zero line upward we take it as a localized 1 (spike-like); otherwise it is 0. Also in this case a non-isochronous synchronization can be observed, however for strong couplings the system tend to synchronize isochronously.

We are interested in the way the defects decay in HC and Rössler. In Fig. 1 we report the space-time series starting from  $t = 0$  as well as from  $t = 500$  (time being in  $\langle IST \rangle$ )

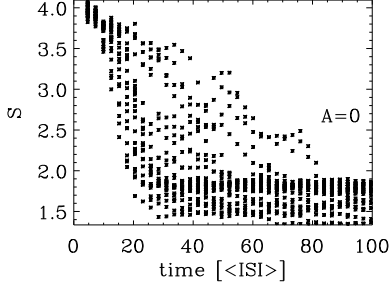


FIG. 3: Entropy  $S$  for 100 coupled HC calculated throughout the initial time slot, starting from different initial conditions, in the absence of external driving and for  $\epsilon = 0.136$ .

or  $T$  units); space is the linear site sequence. For HC with  $\epsilon = 0.1$  in both panels of Fig. 1a we observe the same number of defects on average, thus there is no decay of defects. We obtain the same results also for longer times (not shown here). Defects die out at one site and are born again at another one in course of time, due to the temporal relations between adjacent sites necessary to induce the escape from the saddle region (see the detailed discussion for unidirectional coupling in Ref. [15]). In Rössler the appearance of defects is a global phenomenon: the non zero phase difference between neighboring sites lasts after a long time (left panel in Fig. 1b). Once the array establishes the synchronization line, defects do not reappear again as seen in right panel of Fig. 1b.

We characterize the degree of synchronization in HC and Rössler in terms of two quantities, namely, the number of defects  $N_d$  in a space-time interval corresponding respectively to the 40 sites and  $20\langle ISI \rangle$  and the entropy  $S$ . As the control parameter we take the coupling strength  $\epsilon$ . In HC,  $N_d$  decreases with  $\epsilon$  and goes to zero at the critical coupling strength  $\epsilon_c = 0.13$  (Fig. 1c). In Rössler instead, defects decay monotonically with the coupling strength (Fig. 1d) but no sharp transition is observed. The degree of order of the system is obtained by calculation of the entropy  $S$  from the distributions of the response times  $t_r$  in the time series during or far beyond the initial transient. The entropy  $S$  is defined as

$$S(t) = - \sum_{t_r} p(t_r, t) \ln p(t_r, t) \quad (3)$$

where  $p(t_r, t)$  is the normalized probability distribution of  $t_r$  evaluated at time  $t$ . As done for  $N_d$ , we take for  $p(t_r, t)$  the average over intervals of  $20\langle ISI \rangle$ , starting at  $t$ .

In the case of HC (Fig. 2a)  $S$  has a sharp transition at  $\epsilon = \epsilon_c$ . The transition appears exactly at the critical coupling for which the number of defects goes to zero (Fig. 1c). The transition to synchronization during the transient time has similar characteristics as the synchronization far beyond the transient (see Fig. 2a). Also in this case the sudden decrease of entropy at the critical point is associated with the total disappearance of defects. When the coupling strength is set above  $\epsilon_c$ , defects decay immediately after the coupling is switched on. Indeed, below  $\epsilon_c$  the occurrence of defects hinders syn-

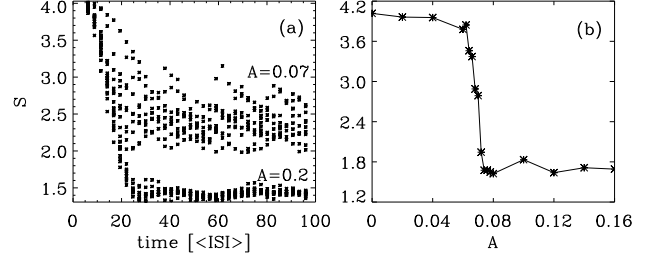


FIG. 4: HC array,  $\epsilon = 0.11$ : (a) Entropy vs time for different amplitudes of an external signal with period  $T = 0.5\langle ISI \rangle$ ; (b) long time entropy vs input amplitude  $A$ , for the same  $T$ .

chronization between sites. The reason is that after the appearance of a defect, the system needs some time to establish synchronization again. Such appearance and disappearance of defects (or equivalently, synchronization and de-synchronization of spikes) is responsible for the lack of synchronization in the system at small coupling strengths. Finally, we checked that the value of the critical coupling is universal for any size of the array. The reason for that is the locality of the bidirectional coupling here considered.

Now we check if the transition occurs also in Rössler. We estimate the entropy  $S$  versus  $\epsilon$  during the initial transient and far beyond it, as in the case of HC. In Fig. 2b the decrease of entropy  $S$  versus  $\epsilon$  is smooth, without sudden changes. It is because defects decay steadily during the time evolution. In this case, once the synchronization is established, defects do not reappear as it happens instead in HC. On the other hand, as shown in Fig. 2b, defects appear during the transient time up to large values of the coupling strength.

Thus, there exists a crucial difference between spike and phase synchronization, namely, in HC defects appear and disappear constantly on average up to  $\epsilon_c = 0.13$ . Beyond  $\epsilon_c$  defects never reappear. On the contrary, in Rössler, there are no critical changes in the defect dynamics, and for non-zero coupling strengths they always decay in the course of the time evolution. The defect dynamics in HC appears as a new feature; in fact in sudden symmetry changes of space extended Ginzburg-Landau type systems transient defects decay always, as shown theoretically [16] and verified experimentally in  $\text{He}^3$  [17] and in nonlinear optics [18]. The difference in defect dynamics in HC and Rössler highlights the existence of the phase transition in the synchronization of the HC systems.

The initial transient is interesting from a neurological point of view, since perception implies synchronization within the short time slot as remarked above. We must then explore a *transient phase transition*, requiring that all sites be synchronized within a limited time following the application of an external signal. We set the coupling strength slightly above the critical value  $\epsilon_c = 0.13$  and measure the entropy  $S$  of the initial time slots ( $t = 0$ ) for different sets of initial conditions. The results are reported in Fig. 3. There are two things to be noticed. First, the way in which the transients differ:

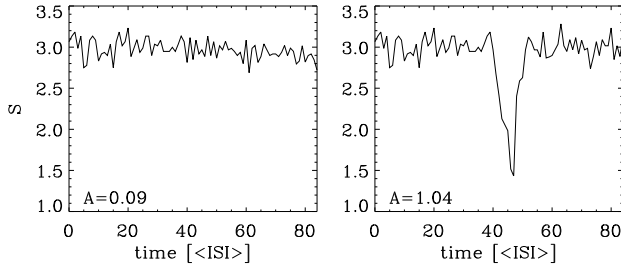


FIG. 5: The entropy in time for HC array perturbed at only one site by a pulse with amplitude  $A = 0.09$  (left panel) and  $A = 0.104$  (right panel). In both cases duration of pulse is  $\Delta t = \langle ISI \rangle$  and the coupling strength  $\epsilon = 0.104$ .

the spread of entropies has a maximum some time after the coupling has been switched on and before the transition to synchronization appears. This deviation is due to the different competition histories between the coupled sites. Second, the existence of different final values of entropy after the transition to synchronization. This difference is determined by the existence of different ordered states (attractors) [19].

From Fig. 3 it appears that the ordered states are reached after  $20\langle ISI \rangle$ . Since feature binding has been related to the gamma band neuronal oscillations, with  $\langle ISI \rangle \sim 25ms$  [2, 4], and a perceptual task requires a few hundred milliseconds [4], such a task has to be over within less than  $30\langle ISI \rangle$ .

In order to model the perceptual task, the transition to synchronization has to be accomplished in the presence of the external driving. Let us consider a periodic signal applied at the first site of the array. We introduce the signal by modulation of parameter  $b_0$  as:  $b = b_0(1 + A \sin 2\pi t/T)$ . Entropy depends on the four parameters  $\epsilon$ ,  $t$ ,  $A$  and  $T$ . Leaving to Ref. [19] a detailed study, here we comment on the following issues. In Fig. 4a, we plot the time behavior of  $S$  for  $\epsilon = 0.11$ , below spontaneous synchronization, and show that an input with period  $T = 0.5\langle ISI \rangle$  yields a synchronized array for  $A = 0.2$ , whereas, for lower  $A$ , the interval 20 to  $80\langle ISI \rangle$  is still at high entropy. In Fig. 4b, we fix  $t = 20\langle ISI \rangle$ ,  $\epsilon = 0.11$ ,  $T = 0.5\langle ISI \rangle$  and plot the dependence of  $S$  on  $A$ . A sharp discontinuity occurs around  $A_c \simeq 0.07$ . Here a novel feature emerges: while each individual system is chaotic and slightly perturbed by the input, the global array behaves as an excitable system, with a sharp transition from a steady high value of entropy to a low one. At variance with previous models which consist of arrays of excitable individuals, usually taken as FitzHugh-Nagumo systems [20], here the excitable behaviour emerges as a collective property of the coupled array. To test the excitable property of an array, we show in Fig. 5 how stimuli of amplitude  $A$  (above  $A_c$ ) and duration  $\Delta t$  induce an excitable collective response. The response has a form of synchronization lasting a finite time; then the array comes back to its previous state.

In summary, we have demonstrated a phase transition in coupled chaotically spiking systems (HC) and characterized this transition in terms of the disappearance of defects at the

critical coupling strength  $\epsilon_c$ . Below  $\epsilon_c$  and in the absence of a stimulus, defects persist in course of time. On the other hand in chaotic non-spiking systems as Rössler, where synchronization regards the phase, and not the spike positions, defects decay at long times even for very small coupling strengths. The important feature of the spiking chaotic systems as HC is that transition to synchronization in response to an external driving is sudden, as it occurs in every day experiences related to recalling the memory of events or objects (we recall a record in a step manner: either we record it or not). We attribute this sudden coherent response to a global excitability property.

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